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SUMMARY

Measuring and inversion of airborne electromagnetic (AEM) data provides one efficient approach to image the subsurface (in this case resistivity) variation. Ideally this should provide decision makers the ability to take informed decisions. In reality, one optimal model is most often found (often smooth and fitting the geophysical data) and used to represent the subsurface. Such deterministic inversion methods rely on implicit model assumptions, representing prior information (a type of regularization information), that may be (but most probable is not) consistent with the actual available information. Further, such methods cannot fully account for the full uncertainty. Hence, it may be impossible to take informed decisions about the subsurface, solely from such one model. Here we present an approach for inversion of AEM data in which geological prior information is independently and explicitly chosen before inversion is carried out. The main benefit of this approach is that a number of subsurface models will be constructed, that will, by construction, be consistent with all information available (both the prior and the data). From such a collection of models detailed uncertainty analysis is possible, and it can be used as the base of decision makers to answer complex questions, without being experts in AEM data or geological modelling. We present a number of examples of using explicit choices of prior information and conclude that the choice of prior information cannot be avoided. If one does not choose a prior model explicitly, it will be chosen implicitly by the choice of inversion algorithm.

Key words: inversion, probabilistic, prior, AEM.

INTRODUCTION

Multiple airborne electromagnetic (AEM) methods exist that can be used to measure an EM response, sensitive to near surface resistivity (down to hundreds of meters depending on the method) e.g. TEMPEST, SkyTEM, and VTEM. Inverse methods can be used to infer information about the distribution of subsurface resistivity (and properties related to resistivity) from observed EM data.

Many different approaches have been considered to solve this inverse problem, that can roughly be regarded as either a 'deterministic' or 'probabilistic' method. The goal of deterministic methods is to locate one, in some sense optimal, resistivity model alone with some uncertainties, that represents observed data, see e.g. Constable et al. (1987) or Menke (2012). The goal of probabilistic methods is to locate a (large) collection models (realizations from a posterior probability density) which ideally represents all available information, Tarantola (2005). In any case, inversion of AEM data is an under-determined inverse problem. Infinitely many models will be able to fit the observed data within its noise. This means that additional prior information must be assumed to solve the inverse problem.

For deterministic inversion methods prior information is provided through `regularization', that can for example control the degree of simplicity/smoothness of the solution model. Most such prior information is *implicitly* chosen through the choice of inversion algorithm. Probabilistic formulated inverse problems require/allow a prior model to be chosen *explicitly*. Below we demonstrate the use of such an explicitly chosen prior model and discuss how it affects resolution and useability as a tool for decision makers.

METHOD AND RESULTS

In a probabilistic formulation of inverse problems, the goal is to describe all available information in one probability distribution, $f_{post}(\mathbf{m})$. Typically, at least two types of information are available. **a**) Information from geophysical data, such as AEM data is quantified through $f_{data}(\mathbf{m}) = L(\mathbf{d}_{obs} - g(\mathbf{m}))$, where $\mathbf{d}_{obs}$ is the observed data and g is a forward mapping operator (solving in this case Maxwell's equations). $f_{data}(\mathbf{m})$ describes the expected noise on the data and is often considered to follow a Gaussian distribution. **b**) '*Prior*' information is quantified through $f_{prior}(\mathbf{m})$, which describes other information about **m** not described by the data. If these two types of information are obtained independently, the 'posterior' probability density that describes the combined information is given by

$$f_{post}(\mathbf{m}) \propto f_{prior}(\mathbf{m})\, f_{data}(\mathbf{m}). \qquad (1)$$

A general approach to extract information about $f_{post}(\mathbf{m})$ is to use sampling methods (e.g. Mosegaard and Sambridge, 2002), where the goal is to generate a set of realizations (in this case resistivity models) that distributed according to $f_{post}(\mathbf{m})$. Many different sampling methods can be used to sample from $f_{post}(\mathbf{m})$ (e.g. Hansen et al., 2016). In the following we use extended Metropolis sampler as described in Hansen et al. (2013).

Inversion of AEM data from Morill Data

As an example we consider a 2D profile of frequency-domain AEM data collected using a helicopter in Nebraska, as described in Minsley et al (2011). Figure 1 shows for reference an inverted 2D resistivity profile obtained using

RES2DINV, which is a linearized least squares type deterministic inversion method, that implicitly relies on Gaussian prior assumptions.

Three different a priori models on the spatial variability is considered, ranging from high to low entropy (i.e. low to high information content). In these cases, the subsurface is parameterized into a 112x151 size pixel model, where each pixel refer to one model parameter, m_i, describing log10-resistivity. Each pixel is of size 1x200m, and the total model is then 22.4 km long and 150 meters deep below the surface.

Case A: Maximum entropy prior. Given the chosen parameterization the simplest prior model, with least information, is the choice of a) no correlation between the model parameters, and b) a uniform distribution of the log-resistivity, here chosen to be $f_A(\mathbf{m}) \propto U[-1,3]$, (i.e. between 0.1 and 1000 ohm-m). Figure 2a shows one realization of this type of prior model.

Case B: Minimum entropy prior. Another prior is chosen to be a three-layer model, where the resistivity within each layer is constant, but known a priori only to be in a certain interval. Further, the depth to the boundary is chosen to be described by multivariate Gaussian along the x-direction (1x451) with a range of 1000 m. The amount of entropy related to such a prior model is much lower than for case A, and we refer to this prior model as a 'minimum entropy' prior, $f_B(\mathbf{m})$. Figure 2b shows one realization of this type of prior model.

Case C: 'realistic choice of prior. This third type of prior considered is one based on available information from the area. It is known that the subsurface is divided into three lithologies, with somewhat overlapping 1D marginal distributions. Is also known that some correlation should be expected between the model parameters, and that this correlation should be along the direction of the surface, and less vertically. A 2D multivariance Gaussian model, $f_C(\mathbf{m})$, is chosen to represent this information, using 1D normal score transformation to ensure the non-Gaussian 1D marginal distribution. Figure 2c shows one realization of this type of prior model.

Case C is the best representation of the available information. It is basd on a Gaussian model, which is known to be a maximum entropy model, beyond the assumed 2-point statistics (described by the covariance). In other words, no information about higher order statistical relations are imposed. This follows good practice of not assuming anything a priori that is not explicitly known. Case A represents an extreme choice in this relation as the only assumed information is that 1D marginal distribution of the log-resistivity is uniform.

The AEM data has been inverted by sampling the posterior distribution using the three defined prior models, $f_A(\mathbf{m})$, $f_B(\mathbf{m})$, and $f_C(\mathbf{m})$. One realization of each set of obtained sample, is shown in Figure 3a-c. Ideally a bunch of such realizations should be inspected. Comparing these realizations of the unconditional realizations in Figure 2 highlights the amount of information contributed by the AEM data.

It is clear that using the high entropy prior, $f_A(\mathbf{m})$, not much information is coming from the data except for a few low resisistivity regions. Also, in most places the resistivity values of neighbouring cells can vary wildly. This is simply due to the fact that the a priori assumptions of no spatial correlation, and the data themselves does not contain information enough to resolve coherent structures in most cases, if they should exist.

Figures 4a-c show the pointwise most probable log-resistivity value (i.e. the mode of the 1D marginal posterior probability density), and highlights that when the resolution is high, the most mode can be an valuable statistic. Note though that the model models cannot be used as a representative model of the subsurface. It is but a 1D statistical measure that may be useful. The actual realizations in Figure 3, represent examples of subsurface variability consistent with all the available information, and can be used for further uncertainty analysis/propagation. For example, Figure 2c and 3c demonstrates how the same kind of variability exist in both the prior and the posterior realization. The only difference is that the location of the coherent regions of resistivity is well defined in multiple realizations of posterior probability in the top region, but close to unconstrained in the bottom part of the model. The variability in the bottom part of the model is not sensitive to the AEM data, and hence just represent the prior. Using the mode model, however, the small scale variability that is chosen to exist a priori, is filtered out, and hence this model cannot be used to any uncertainty analysis propagation.

Figures 4a-c show the Kullback-Leibler (KL) distance between the posterior and prior distribution at each model parameter. It is a measure how different the 1D marginal prior and posterior are. It is a measure of how much information is gained by adding the data, and hence can be used as a type of resolution analysis. A KL-distance of zero indicates that no information has been added by using the data, as the 1D posterior distribution is the same as the 1D prior distribution. Note though that even though the KL distance may be zero, a model parameter may be perfectly resolved, as in case B, where the prior assumption is that there is a constant low resistive layer at the bottom. In this case the resolution stems from a combination of very strong prior information at depth, that needs only very little information in the top of the model to be completely constrained. In this case not much information is gained at the bottom of the model, as indicated by the KL distance in Figure 4, but the resistivity at depth is still well resolved due to the prior.

This off course emphasize the crucial role of the prior. If one wish to assume as little as possible, as in $f_B(\mathbf{m})$, then one run the risk of introducing small scale variability into the solutions models (posterior realizations), that is geologically unrealistic. This means that any further propagation of uncertainty, such as through flow modelling, may provide erroneous results. Which again means that any risk assessment may turn out erroneous.

On the other hand, of too much information is assumed, as in $f_B(\mathbf{m})$, then the AEM data can still be fitted ,and one gets a apparently well resolved model, where realizations from the posterior are very similar. This unrealistically small posterior variability is though mostly determined by the use of a strong (and in this case probably wrong) prior. Again, any subsequent risk assessment may provide erroneous results. Thus, posterior analysis using both $f_A(\mathbf{m})$ and $f_B(\mathbf{m})$ is thus hampered by the choice of a wrong/inconsistent prior.

In case constructing a prior model actually representing the information available, as $f_C(\mathbf{m})$, it is possible to solve the inverse problem to generate a collection of resistivity models that reflect both realistic a priori information, and that is consistent with the AEM data. The variability of such a set of models, can then be mapped into other domains (such as computing the volume of a potential reservoir with uncertainty).

Conclusions

Inversion of AEM data leads to a severely underdetermined inverse problem, which means that the choice of prior information is essential to allow any realistic uncertainty analysis. The choice of prior information cannot be avoided. If available prior information can be described in a probabilistic manner, then using a probabilistic formulation of inverse problems, it can be combined with information from AEM data, to provide a set of subsurface resistivity models, that can be used as a base for decision makers. Informed decisions can then be taken consistent with both available geophysical data, and available geological information quantified as prior information.

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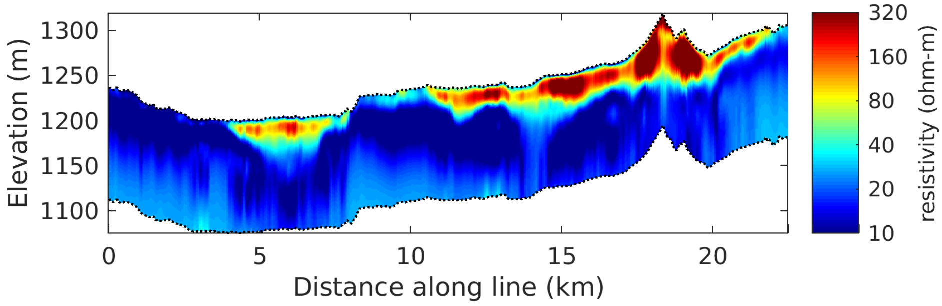

Figure 1, Linearized least squares inversion along AEM profile

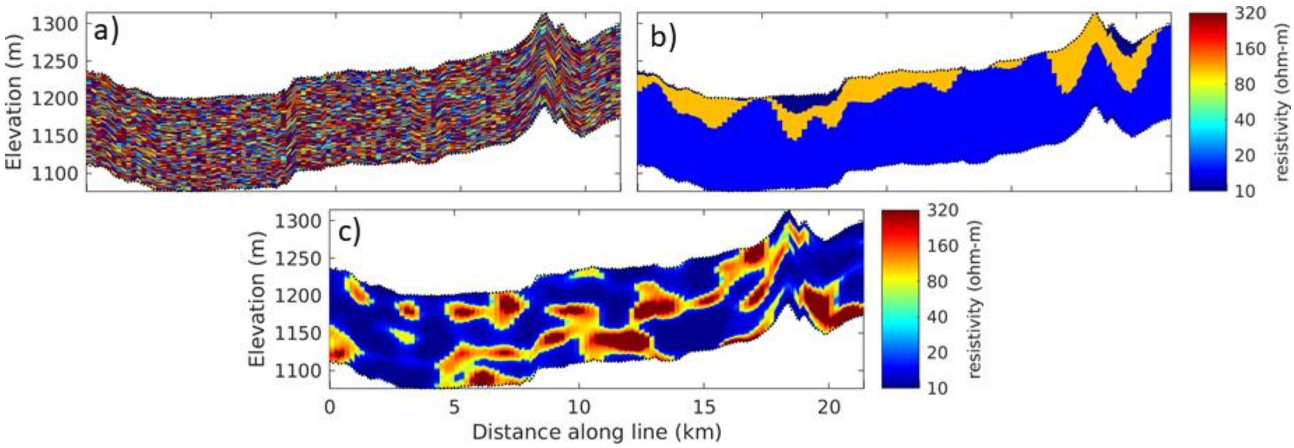

Figure 2, One realizations from each of the the priors, f_A(m), f_B(m), and f_C(m).

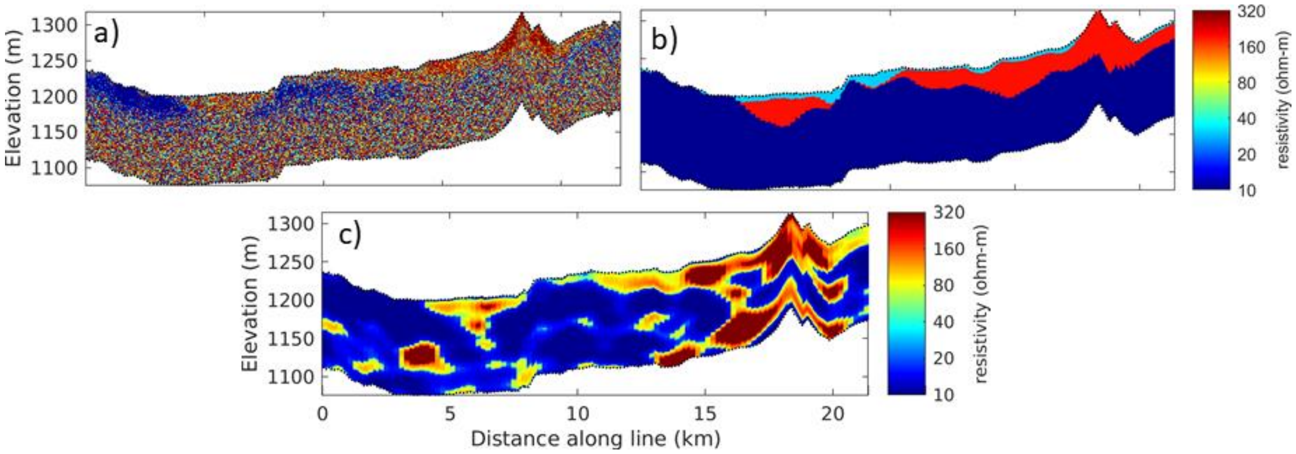

Figure 3, One realizations from the posterior, for case A, B, and C.

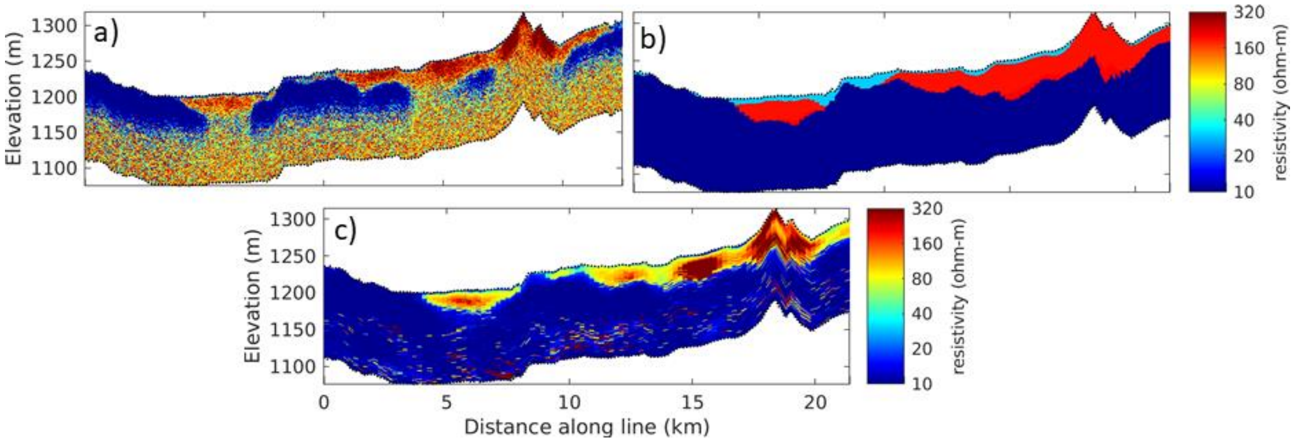

Figure 4, The pointwise most probable resistivity (the mode) of 1D marginal posterior probability density for case A, B, and C.

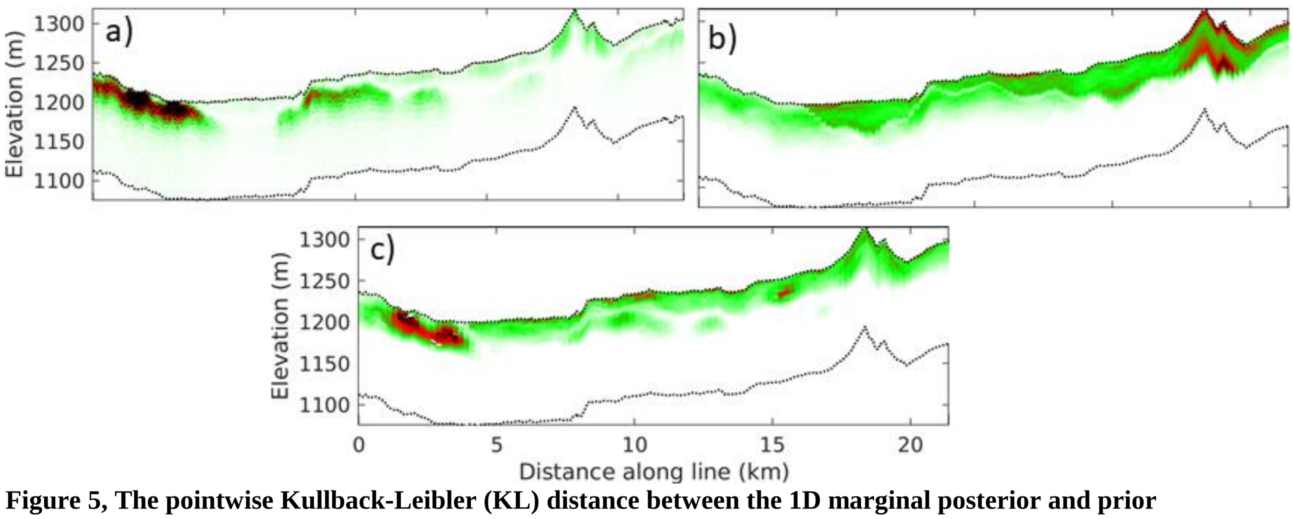

Figure 5, The pointwise Kullback-Leibler (KL) distance between the 1D marginal posterior and prior distribution. Black indicates large distance, and white indicates small distance. a) KL($f_{Post,A}$(m), f_A(m)). b) KL($f_{Post,B}$(m), f_B(m)). c) KL($f_{Post,C}$(m), f_C(m)).